

Introduction to Robust and Distributionally Robust Optimization

BiGSEM Mini Course: 5×90 minutes, May 13–15, 2024

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Last Update: March 19, 2024

Course Overview and Objectives

Robust optimization is an optimization technique for making effective decisions under uncertain situations. Originating from a series of studies by Ben-Tal and Nemirovski [BTN98, BTN99, BTN00], this field has long been recognized as an important area of mathematical optimization due to its practicality. In recent years, a new approach called “distributionally robust optimization,” which is an extension of robust optimization, has attracted much attention, and research in this field has been actively conducted.

This course aims to provide an overview of the modeling techniques used in robust and distributionally robust optimization. We will start with the fundamentals of mathematical optimization and introduce the theory of convex optimization problems. Specifically, the lecture will focus on linear optimization and conic optimization problems, as well as their duality theory, which plays an important role in robust optimization. The course then moves on to the modeling of robust optimization, which aims to optimize the worst-case performance where uncertain parameters lie in a certain set (*uncertainty set*). Finally, we will further extend robust optimization and introduce the distributionally robust optimization, which addresses uncertainty of probability distributions of parameters.

Prerequisites

This course assumes that students are familiar with basic linear algebra. No prior knowledge of mathematical optimization is required.

Evaluation

Your final grade will be determined by homework assignments designed to test your understanding of the course topics. Details will be provided during the course.

Course Schedule

Date	Lecture	Topic	References
May 13	1	Introduction to Mathematical Optimization: Linear Optimization	[LY08, Van20]
	2	Convex and Conic Optimization	[BV04, WSV12]
May 14	3*	Economic Seminar: Mixed-integer Semidefinite Optimization and Its Applications to Portfolio Optimization	[KTN23, KT19]
	4	Modeling of Robust Optimization	[BTEGN09, BTN98]
May 15	5	Modeling of Distributionally Robust Optimization	[RM19, DY10, MEK17]

* The third lecture will be held as a stand-alone Economic Seminar. Its contents are self-contained, but they include related topics of robust and distributionally robust optimization.

References

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- [KTN23] Ken Kobayashi, Yuichi Takano, and Kazuhide Nakata. Cardinality-constrained distributionally robust portfolio optimization. *European Journal of Operational Research*, 309(3):1173–1182, 2023.
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